REMEDIAL MATHEMATICS BP106RMT

Unit-I

Partial Fractions

UNIT-1 PARTIAL FRACTION

- An algebraic fraction can be broken down into simpler parts known as “partial fractions“. Consider an algebraic fraction, (3x+5)/(2x2 -5x-3). This expression can be split into simple form like (2)/(x-3) – (1)/(2x+1).
- The simpler parts [(2)/(x-3)]-[(1)/(2x+1)] are known as partial fractions. This means that the algebraic expression can be written in the form of: (3x+5)/(2x2 -5x-3) = ((2)/(x-3))-((1)/(2x+1))
- Note: The partial fraction decomposition only works for the proper rational expression (the degree of the numerator is less than the degree of the denominator). In case, if the rational expression is in improper rational expression (the degree of the numerator is greater than the degree of the denominator), first do the division operation to convert into proper rational expression. This can be achieved with the help of polynomial long division method.

PARTIAL FRACTION FORMULA

- The procedure or the formula for finding the partial fraction is:
- 1. While decomposing the rational expression into the partial fraction, begin with the proper rational expression.
- 2. Now, factor the denominator of the rational expression into the linear factor or in the form of irreducible quadratic factors (Note: Don’t factor the denominators into the complex numbers).
- 3. Write down the partial fraction for each factor obtained, with the variables in the numerators, say A and B.
- 4. To find the variable values of A and B, multiply the whole equation by the denominator.
- 5. Solve for the variables by substituting zero in the factor variable. 6. Finally, substitute the values of A and B in the partial fractions

PARTIAL FRACTIONS FROM RATIONAL FUNCTIONS

Any number which can be easily represented in the form of p/q, such that p and q are integers and $q \neq 0$ is known as a rational number. Similarly, we can define a rational function as the ratio of two polynomial functions P(x) and Q(x), where P and Q are polynomials in x and $Q(x) \neq 0$. A rational function is known as proper if the degree of P(x) is less than the degree of Q(x); otherwise, it is known as an improper rational function. With the help of the long division process, we can reduce improper rational functions to proper rational functions. Therefore, if P(x)/Q(x) is improper, then it can be expressed as:

$P(x)Q(x)=A(x)+R(x)Q(x)$ Here, A(x) is a polynomial in x and R(x)/Q(x) is a proper rational function

We know that the integration of a function f(x) is given by F(x) and it is represented by: $\int f(x)dx = F(x) + C$ Here R.H.S. of the equation means integral of f(x) with respect to x.

PARTIAL FRACTION OF IMPROPER FRACTION

- An algebraic fraction is improper if the degree of the numerator is greater than or equal to that of the denominator. The degree is the highest power of the polynomial. Suppose, m is the degree of the denominator and n is the degree of the numerator. Then, in addition to the partial fractions arising from factors in the denominator, we must include an additional term: this additional term is a polynomial of degree $n - m$
- . Note:
- A polynomial with zero degree is K, where K is a constant . A polynomial of degree 1 is $Px + Q$ A polynomial of degree 2 is $Px2+Qx+K$

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$
	• where $x^2 + bx + c$ cannot be factorised further	

Partial Fractions Examples and Solutions

- Type: 1
- **1) Solve the question given below using the concept of partial fractions.**
- Resolve $\frac{7x-25}{(x-3)(x+4)}$ into partial fractions.

LOGARITHMS

- The logarithmic function is an inverse of the exponential function.
- It is defined as:
- y=logax, if and only if x=ay ; for x>0, a>0, and a≠1.
- Natural logarithmic function: The log function with base e is called natural logarithmic function and is denoted by loge.
- f(x) = logex
- The questions of logarithm could be solved based on the properties, given below Product rule:
- logb MN = logb M + logb N
- Quotient rule: logb M/N = logb M – logb N
- Power rule: logb Mp = P logb M
- Zero Exponent Rule: loga 1 = 0
- Change of Base Rule: logb (x) = ln x / ln b or logb (x) = log10 x / log10 b

FUNCTIONS

- DEFINITION
- Function (mathematics) is defined as if each element of set A is connected with the elements of set B, it is not compulsory that all elements of set B are connected; we call this relation as function.
- f: A → B (f is a function from A to B

Types of function:

- One-one Function or Injective Function :
- If each elements of set A is connected with different elements of set B, then we call this function as Oneone function
- Many-one Function :
- If any two or more elements of set A are connected with a single element of set B, then we call this function as Many one function.

Onto function or Surjective function :

- Function f from set A to set B is onto function if each element of set B is connected with set of A elements.
- Into Function :
- Function f from set A to set B is Into function if at least set B has a element which is not connected with any of the element of set A.

Constant Functon

- A function f: $A \rightarrow B$ is a constant function if the range of f contains only one element.
- $f(x)=4$
- Identity Function:
- Let A be a non – empty set
- then
- f: $A \rightarrow A$ defined by $f(x)=x$, called the identity function on A